Frame based kernel methods for hyperspectral imagery data

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Recent Advances in Harmonic Analysis and Elliptic Partial Differential Equations

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1 Hyperspectral imagery data
   - Introduction to hyperspectral imagery data
   - Endmembers

2 The algorithm
   - Kernel eigenmap methods
   - Frames

3 Results
   - The Urban data set
   - Classification results
Outline

1. Hyperspectral imagery data
   - Introduction to hyperspectral imagery data
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3. Results
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Introduction to hyperspectral imagery data

The algorithm

Results

Endmembers

Color image

Hyperspectral imagery data

Frame based kernel methods for hyperspectral imagery data
Hyperspectral imagery data

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Frame based kernel methods for hyperspectral imagery data
Hyperspectral data cube
Hyperspectral imagery (HSI) data is characterized by the narrowness and contiguous nature of the measurements. HSI data sets are spectrally overdetermined, and thus provide ample spectral information to distinguish between spectrally similar (but unique) materials.

HSI data sets can be useful for the following purposes:
- target detection
- material classification
- material identification
- mapping details of surface properties
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Endmember illustration
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Assume our HSI data set is an $N_1 \times N_2 \times D$ cube.

- $N_1$, $N_2$ spatial dimensions; $N = N_1N_2$ pixels.
- $D$ is the spectral dimension; so $D$ wavelengths measured.

Let $X = \{x_i\}_{i=1}^{N} \subset \mathbb{R}^D$ denote the pixel vectors of the HSI data set in list form.

**Definition**

Endmembers are a collection of a scene’s constituent spectra. If $E = \{e_i\}_{i=1}^{s}$ are endmembers, then the linear mixture model is

$$x_i = \sum_{j=1}^{s} \alpha_{i,j}e_j + N_{x_i}, \quad \forall x_i \in X,$$

where $N_{x_i}$ is a noise vector.
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We have two goals:

1. Map the high dimensional HSI data set $X$ to an appropriate low dimensional space $Y$.
2. Represent the low dimensional space $Y$ for the purposes of material classification.

We achieve these goals via two existing mathematical theories:

1. We use kernel eigenmap methods to determine the space $Y$.
2. We represent $Y$ with an overcomplete endmember set $\Phi$, also known as a frame.
Introduction to kernel eigenmap methods

Given a high dimensional data set $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^D$, we assume $X$ lies on a low dimensional manifold $M^d$ ($d < D$).

Kernel eigenmap methods map the vectors in $X$ to $d$-dimensional coordinates $Y = \{y_i\}_{i=1}^N \subset \mathbb{R}^d$ that preserve the underlying geometry of $M^d$. 
Introduction to kernel eigenmap methods

- Given a high dimensional data set \( X = \{x_i\}_{i=1}^{N} \subset \mathbb{R}^D \), we assume \( X \) lies on a low dimensional manifold \( M^d \) \((d < D)\).

- Kernel eigenmap methods map the vectors in \( X \) to \( d \)-dimensional coordinates \( Y = \{y_i\}_{i=1}^{N} \subset \mathbb{R}^d \) that preserve the underlying geometry of \( M^d \).
The main components of kernel eigenmap methods are:

- Construction of an $N \times N$ symmetric, positive semi-definite kernel $K$,

$$K_{i,j} = K(x_i, x_j).$$

- Diagonalization of $K$ and then choosing $d \leq D$ significant orthogonal eigenvectors of $K$, denoted by $v_1, \ldots, v_d$.

- Map each $x_i \in X$ to the $d$-dimensional vector $y_i$ given by:

$$y_i = (v_1(i), \ldots, v_d(i)).$$
Frame theory

Definition

1. \( \Phi = \{\varphi_i\}_{i=1}^s \) is a frame for \( \mathbb{R}^d \) if there exists \( A, B > 0 \) such that

\[
A \|y\|^2 \leq \sum_{i=1}^{s} |\langle y, \varphi_i \rangle|^2 \leq B \|y\|^2, \quad \forall y \in \mathbb{R}^d.
\]

2. For a frame \( \Phi = \{\varphi_i\}_{i=1}^s \), the frame operator \( S : \mathbb{R}^d \rightarrow \mathbb{R}^d \) is

\[
S(y) = \sum_{i=1}^{s} \langle y, \varphi_i \rangle \varphi_i.
\]
We wish to use a data dependent frame $\Phi = \{\varphi_i\}_{i=1}^s$ to represent the reduced dimension space $Y = \{y_i\}_{i=1}^N \subset \mathbb{R}^d$:

$$y_i = \sum_{j=1}^s c_{i,j} \varphi_j, \quad \forall y_i \in Y.$$ 

Possible frame construction algorithms:
- existing endmember algorithms
- modified frame potential [Benedetto, Fickus; 2003]

Possible frame coefficients:
- $c_{i,j} = \langle y_i, S^{-1}(\varphi_j) \rangle$
- $c_{i,.} = \arg \min_c \|c\|_{\ell^1}$ subject to $\Phi c = y_i$
Why use frames?

- Traditional endmembers may be mixtures of many prominent features.
- Overestimating the number of classes allows for flexibility in representing mixtures and pure elements.
- Empirical evidence suggests that distinct classes are not orthogonal to each other. Unlike the orthogonal eigenvectors of $K$, frame elements need not be orthogonal.
Review of algorithm

\[ K \subset \mathbb{R}^N \]
\[ X \subset \mathbb{R}^D \]
\[ \{c_{i,j}\} \subset \mathbb{R}^s \]
\[ Y \subset \mathbb{R}^d \]

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Urban

Figure: HYDICE Copperas Cove, TX – http://www.tec.army.mil/Hypercube/
Urban classes

22 classes in the Urban data set, including:

- Dark asphalt
- Vegetation: grass
- Soil 1
- Soil 2
- Soil 3 (dark)
- Roof: Walmart
Overview of classification results
Sample frame coefficients

[Images of sample frame coefficients with color scales]
Dark asphalt

(e) Urban

(f) Dark Asphalt
Hyperspectral imagery data
The algorithm
Results

The Urban data set
Classification results

Light asphalt

(a) Urban
(b) Light asphalt
Concrete

(a) Urban

(b) Concrete

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Vegetation: pasture

(a) Urban

(b) Vegetation: pasture
Vegetation: grass

(a) Urban

(b) Vegetation: grass
Vegetation: trees

(a) Urban

(b) Vegetation: trees
Soil 1

(a) Urban

(b) Soil 1
Soil 2

(a) Urban

(b) Soil 2
Soil 3 (dark)

(a) Urban

(b) Soil 3 (dark)
Hyperspectral imagery data

The algorithm

Results

The Urban data set

Classification results

Roof: Walmart

(a) Urban

(b) Roof: Walmart
Roof: A

(a) Urban

(b) Roof: A
Hyperspectral imagery data
The algorithm
Results
The Urban data set
Classification results

Roof: B

(a) Urban

(b) Roof: B
Roof: light gray

(a) Urban

(b) Roof: light gray
Roof: dark brown

(a) Urban

(b) Roof: dark brown
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Roof: school

(a) Urban

(b) Roof: school

Frame based kernel methods for hyperspectral imagery data
Roof: bright

(a) Urban

(b) Roof: bright
Roof: blue green

(a) Urban

(b) Roof: blue green
Tennis court

(a) Urban

(b) Tennis court

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Pool water

(a) Urban

(b) Pool water

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Frame based kernel methods for hyperspectral imagery data
Frame based kernel methods for hyperspectral imagery data

(a) Urban

(b) Shaded vegetation
Shaded pavement

(a) Urban  
(b) Shaded pavement
Thank you for your time.

http://www.math.umd.edu/~hirn/