

Math 414: Linear Algebra II, Fall 2015

Homework 7

Due: November 6, 2015 by 5:00 PM

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All problems are taken from *Linear Algebra Done Right*, 3rd Edition.

- **Exercises 6.C:** 5, 7, 8, 9, 10, 12, 14

Hints:

- 7: Try $U = \text{range}P$.
- 8: You may need to use #6 from 6.A (HW 5) at some point.
- 14 (a): Here is a general idea of how to solve this problem. $U^\perp \subset C_{\mathbb{R}}([-1,1])$. So let $g \in C_{\mathbb{R}}([-1,1])$ with g not the zero function; we want to show that g can't possibly be in U^\perp . This would mean that for each $g \in C_{\mathbb{R}}([-1,1])$, with g not identically zero, there exists some $f \in U$ such that $\langle f, g \rangle = \int_{-1}^1 f(x)g(x) dx \neq 0$.

In order to find such an f , you can use the following fact about continuous functions. Suppose $g \in C_{\mathbb{R}}([-1,1])$ and $g \neq 0$. Because g is continuous on $[-1,1]$ and $g \neq 0$, there exists an interval $(a,b) \subset [-1,1]$ with $a < b$ and $0 \notin (a,b)$ such that g is positive everywhere on (a,b) or g is negative everywhere on (a,b) .

Instructions: You are welcome to *discuss* the homework problems with your peers in the course, but you are *never* allowed to copy each other's solutions. You must write your own solution yourself. Hand in a *clean* and *carefully written* version of each of your solutions.