Math 414: Linear Algebra II, Fall 2015
Homework 8

Due: November 13, 2015 in class

Matthew Hirn

November 2, 2015

All problems are taken from *Linear Algebra Done Right*, 3rd Edition.

- **Exercises 7.A:** 2, 6, 15, 17, 19, 21

Hints:

- 15: First compute T^*.
- 17: Start by proving $\operatorname{null} T^k = \operatorname{null} T$ by showing $\operatorname{null} T \subset \operatorname{null} T^k$ and $\operatorname{null} T^k \subset \operatorname{null} T$. The first inclusion should not be hard. For the second inclusion, let $v \in \operatorname{null} T^k$. Show that this implies $v \in \operatorname{null} T^{k-1}$. Keep repeating your argument so that you get:

  $$v \in \operatorname{null} T^k \Longrightarrow v \in \operatorname{null} T^{k-1} \Longrightarrow v \in \operatorname{null} T^{k-2} \Longrightarrow \cdots \Longrightarrow v \in \operatorname{null} T$$

  Now use the fact that $\operatorname{null} T^k = \operatorname{null} T$ (along with some other stuff) to prove that $\operatorname{range} T^k = \operatorname{range} T$.

Instructions: You are welcome to *discuss* the homework problems with your peers in the course, but you are *never* allowed to copy each other's solutions. You must write your own solution yourself. Hand in a *clean* and *carefully written* version of each of your solutions.