Math 414: Linear Algebra II, Fall 2015
Midterm 2

November 23, 2015

<u>NAME:</u>

This is a **closed book single author exam**. Use of books, notes, or other aids is *not* permissible, nor is collaboration with any of your fellow students.

You must **prove, justify, or explain** all of your assertions.

This midterm is out of 100 **points**.

Please write your **name** above, and at the **top** of each subsequent page.

QUESTION 1:

Suppose V is a finite dimensional inner product space and $T \in \mathcal{L}(V)$.

(a) [10 points] Let v be an eigenvector of T with eigenvalue $\lambda \in \mathbb{F}$. Suppose $|\lambda| < 1$. Prove that for all $\epsilon > 0$, there exists a positive integer m such that $\|T^m v\| \leq \epsilon \|v\|$.

(b) [20 points] Suppose T is a self-adjoint operator such that if $\lambda \in \mathbb{F}$ is an eigenvalue of T, then $|\lambda| < 1$. Prove that for all $\epsilon > 0$, there exists a positive integer m such that $\|T^m v\| \leq \epsilon \|v\|$ for all $v \in V$.

QUESTION 2:

Let V, W be finite dimensional inner product spaces over the field $\mathbb{F}$. Let $T \in \mathcal{L}(V, W)$.

(a) [10 points] Prove that $T^*T \in \mathcal{L}(V)$ and $TT^* \in \mathcal{L}(W)$ are a positive operators.

(b) [20 points] Let $\epsilon > 0$ and let $I \in \mathcal{L}(V)$ be the identity operator on V. Prove that $T^*T + \epsilon I \in \mathcal{L}(V)$ is invertible.

(c) [20 points] Let $\lambda \in \mathbb{F}$, $\lambda \neq 0$. Show that λ is eigenvalue of T^*T if and only if λ is an eigenvalue of TT^*. Furthermore, show that

$$\dim E(\lambda, T^*T) = \dim E(\lambda, TT^*)$$

Question 3:

[20 points] Let $\mathbb{C}^n$ be endowed with the standard inner product, i.e., for $w = (w_1, \ldots, w_n) \in \mathbb{C}^n$ and $z = (z_1, \ldots, z_n) \in \mathbb{C}^n$,

$$\langle w, z \rangle = \sum_{k=1}^{n} w_k \bar{z}_k$$

For each $n = 1, 2, \ldots$, give an example of a rigid motion $f : \mathbb{C}^n \to \mathbb{C}^n$ such that $f(0) = 0$ and f is not linear.

Scratch Paper

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