

Math 414: Linear Algebra II, Fall 2015

Rigid Motion Practice

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Let \mathbb{C} be the complex plane with the standard inner product, i.e.,

$$\langle w, z \rangle = w\bar{z} \implies \|z\| = |z|$$

(a) [easier] Show that the functions $f : \mathbb{C} \rightarrow \mathbb{C}$ and $g : \mathbb{C} \rightarrow \mathbb{C}$, defined as:

$$\left. \begin{aligned} f(z) &= a + \lambda z \\ g(z) &= a + \lambda \bar{z} \end{aligned} \right\} \quad a, \lambda \in \mathbb{C}, \quad |\lambda| = 1,$$

are rigid motions.

(b) [harder] Show that all rigid motions $h : \mathbb{C} \rightarrow \mathbb{C}$ must be of the form of f or g from part (a).