Math 414: Linear Algebra II, Fall 2015
Rigid Motion Practice

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Let \( \mathbb{C} \) be the complex plane with the standard inner product, i.e.,

\[ \langle w, z \rangle = w \bar{z} \implies \|z\| = |z| \]

(a) [easier] Show that the functions \( f : \mathbb{C} \to \mathbb{C} \) and \( g : \mathbb{C} \to \mathbb{C} \), defined as:

\[
\begin{align*}
f(z) &= a + \lambda z \\
g(z) &= a + \lambda \bar{z}
\end{align*}
\]

\( a, \lambda \in \mathbb{C}, \ |\lambda| = 1, \)

are rigid motions.

(b) [harder] Show that all rigid motions \( h : \mathbb{C} \to \mathbb{C} \) must be of the form of \( f \) or \( g \) from part (a).