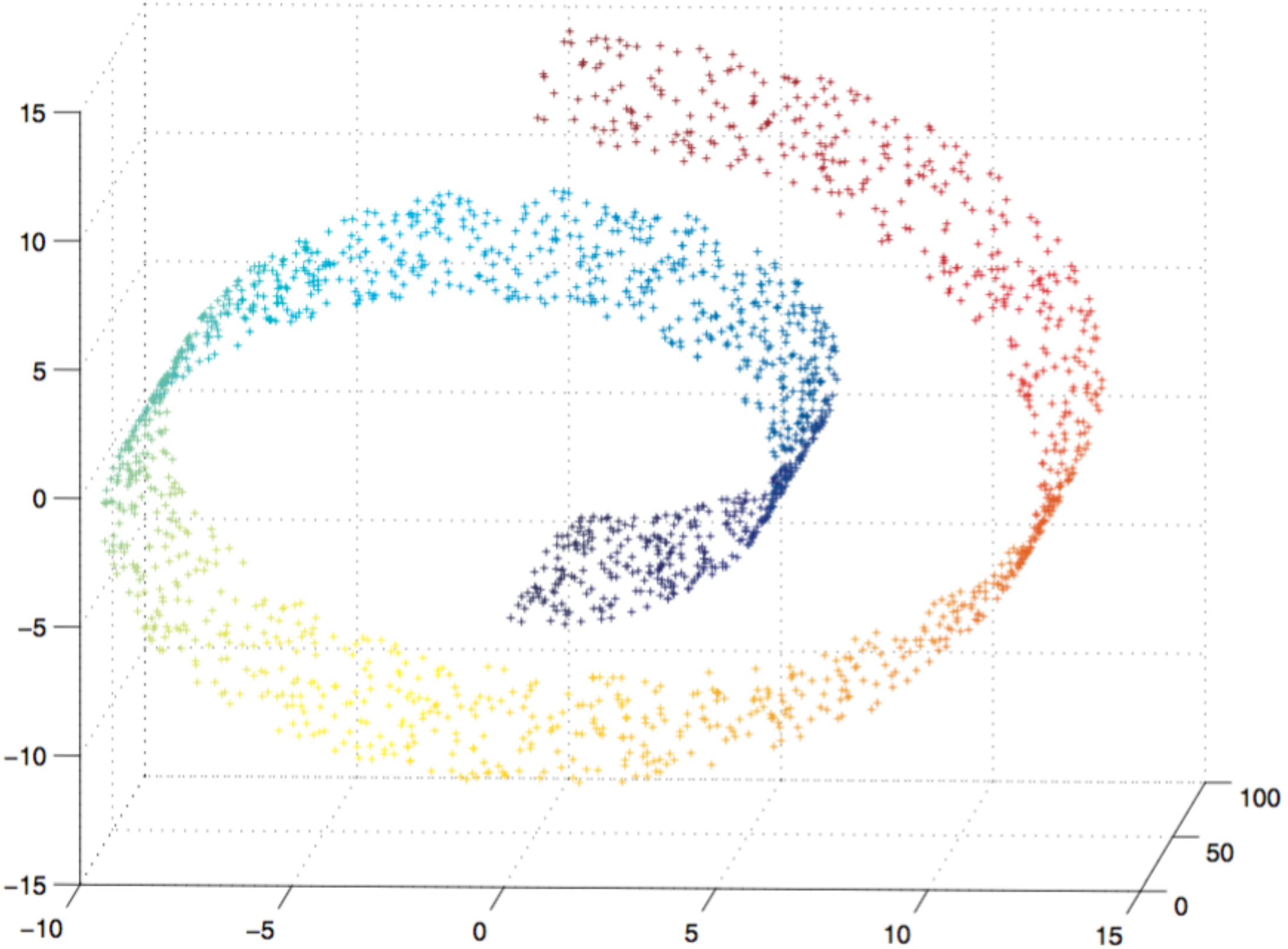
CMSE 820: Mathematical Foundations of Data Science

Lecture 24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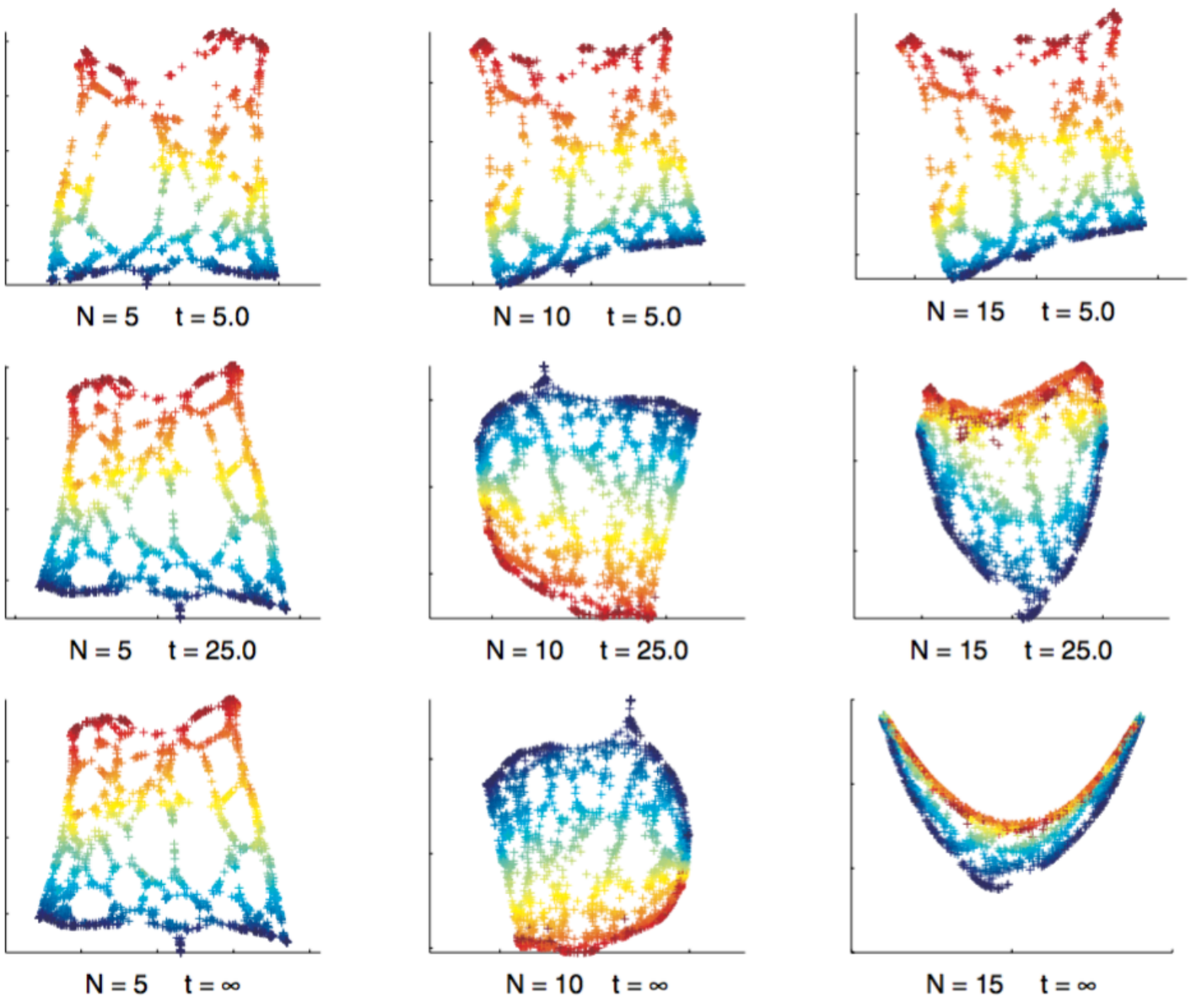


Figure 2: Two-dimensional representations of the swiss roll data, for different values of the number of nearest neighbors N and the heat kernel parameter t. $t = \infty$ corresponds to the discrete weights.

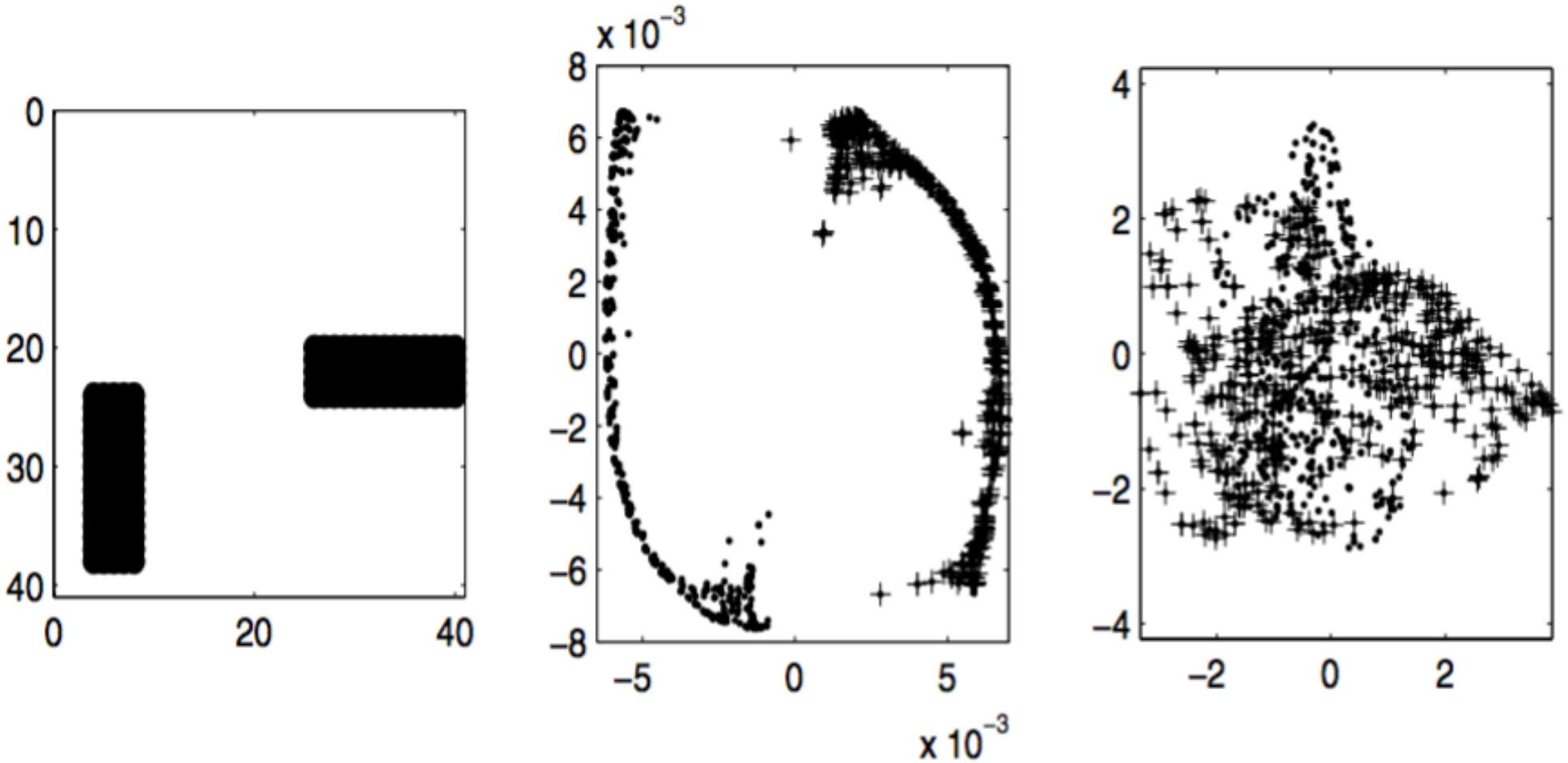


Figure 3: (Left) A horizontal and a vertical bar. (Middle) A two-dimensional representation of the set of all images using the Laplacian eigenmaps. (Right) The result of PCA using the first two principal directions to represent the data. Blue dots correspond to images of vertical bars, and plus signs correspond to images of horizontal bars.

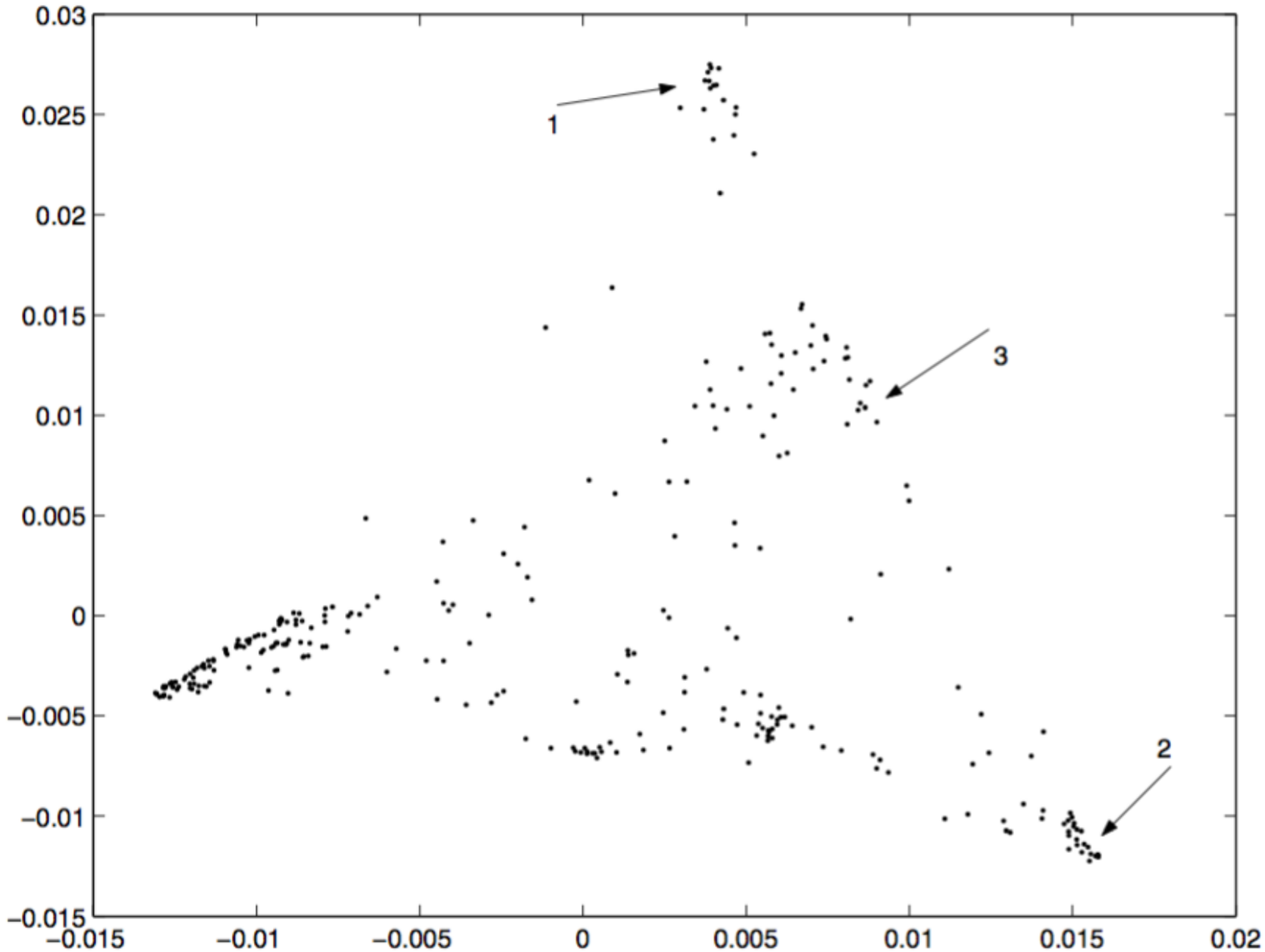


Figure 4: The 300 most frequent words of the Brown corpus represented in the spectral domain.

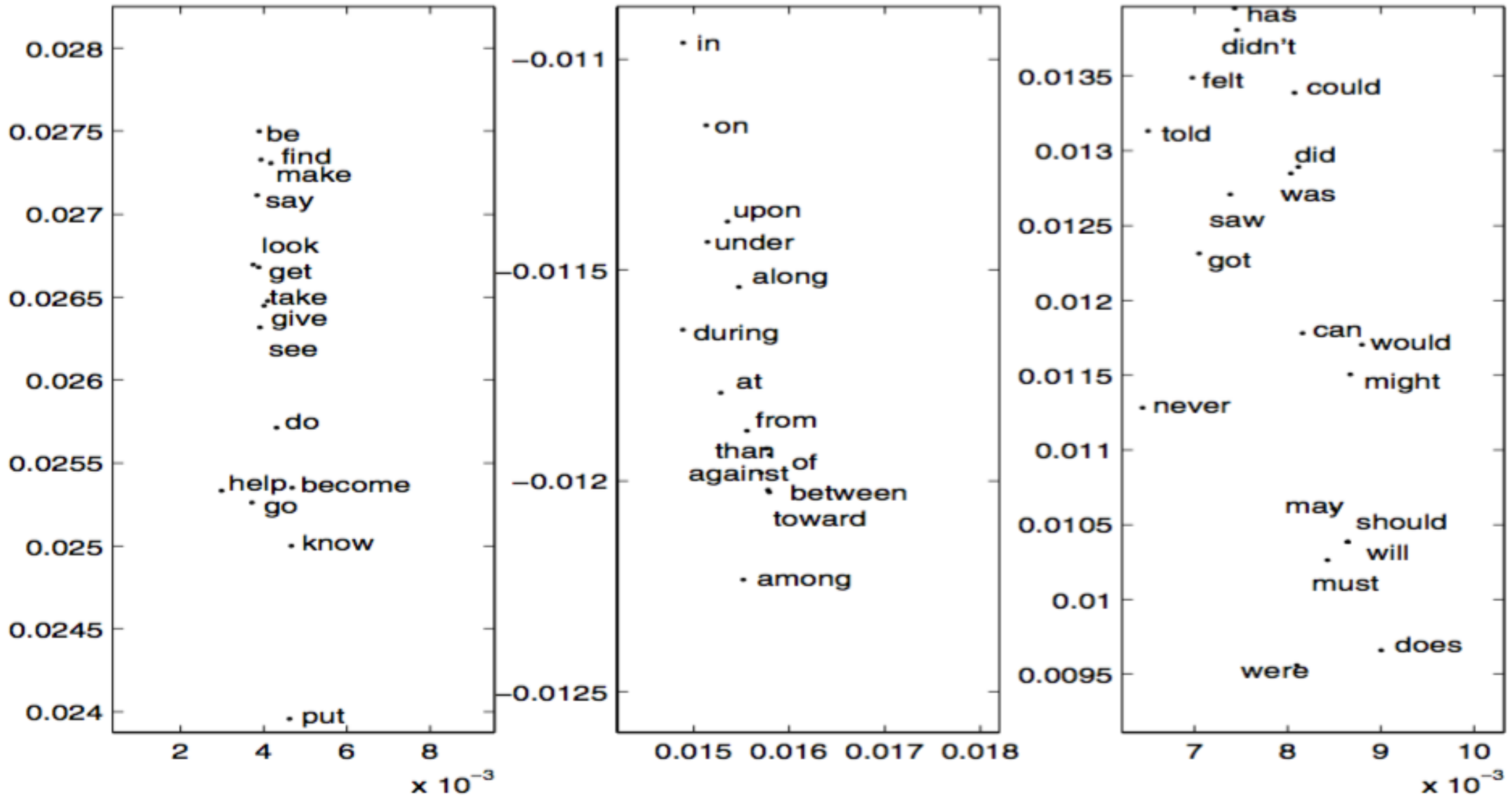


Figure 5: Fragments labeled by arrows: (left) infinitives of verbs, (middle) prepositions, and (right) mostly modal and auxiliary verbs. We see that syntactic structure is well preserved.

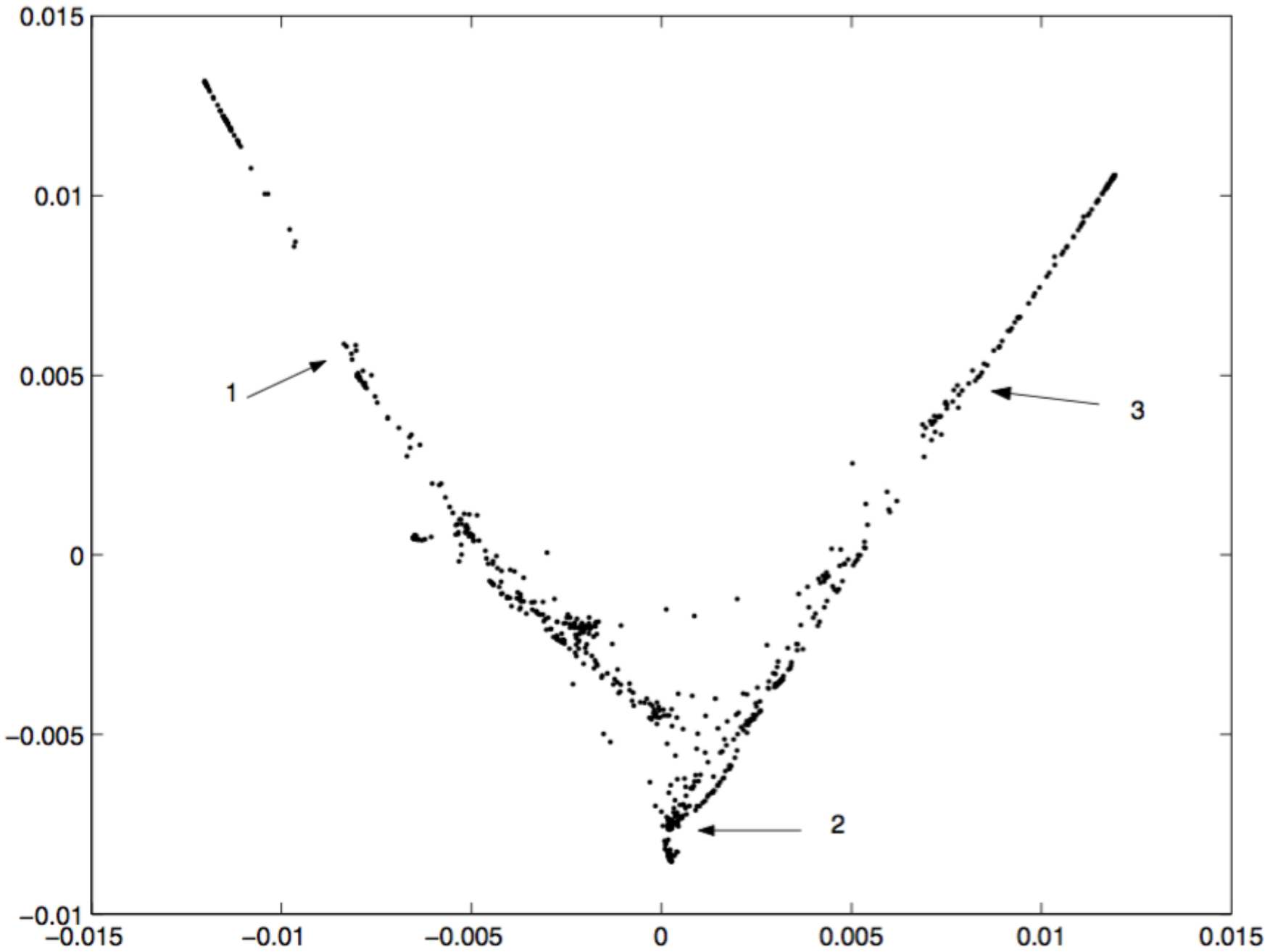


Figure 6: The 685 speech data points plotted in the two-dimensional Laplacian spectral representation.

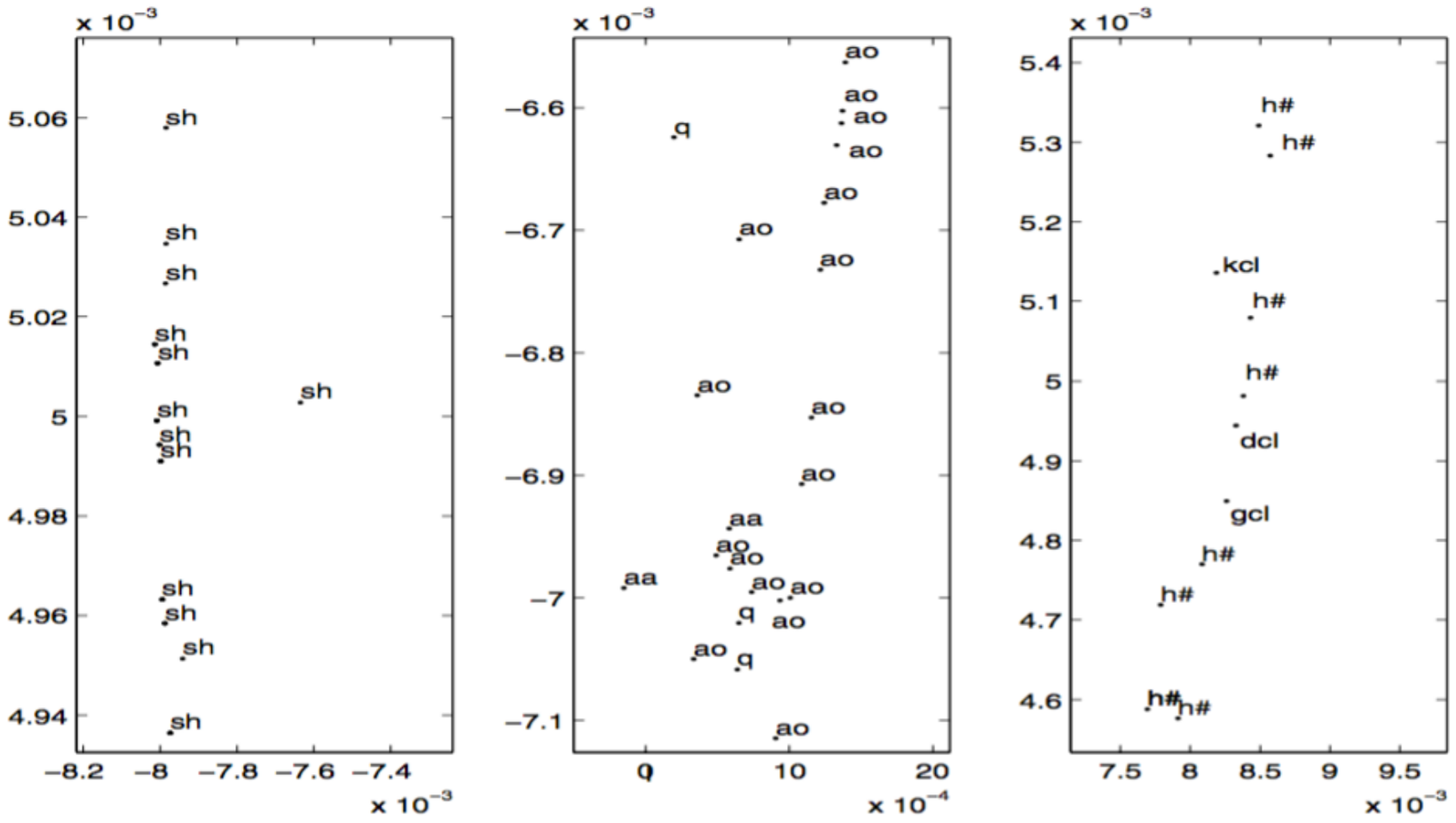


Figure 7: A blowup of the three selected regions corresponding to the arrows in Figure 6. Notice the phonetic homogeneity of the chosen regions. The data points corresponding to the same region have similar phonetic identity, though they may (and do) arise from occurrences of the same phoneme at different points in the utterance. The symbol *sh* stands for the fricative in the word *she*; *aa* and *ao* stand for vowels in the words *dark* and *all*, respectively; *kcl*, *dcl*, and *gcl* stand for closures preceding the stop consonants *k*, *d*, *g*, respectively. *h#* stands for silence.

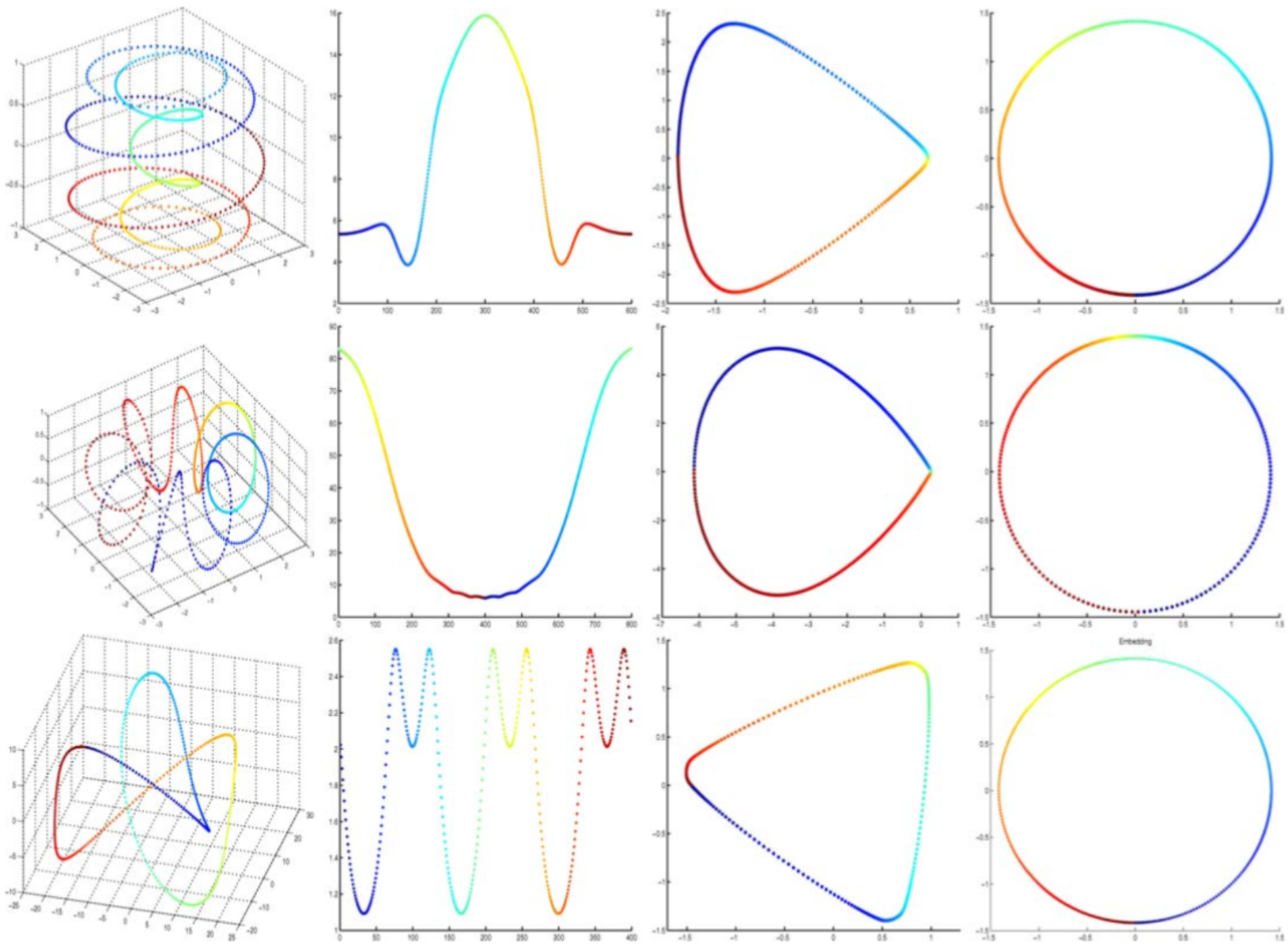
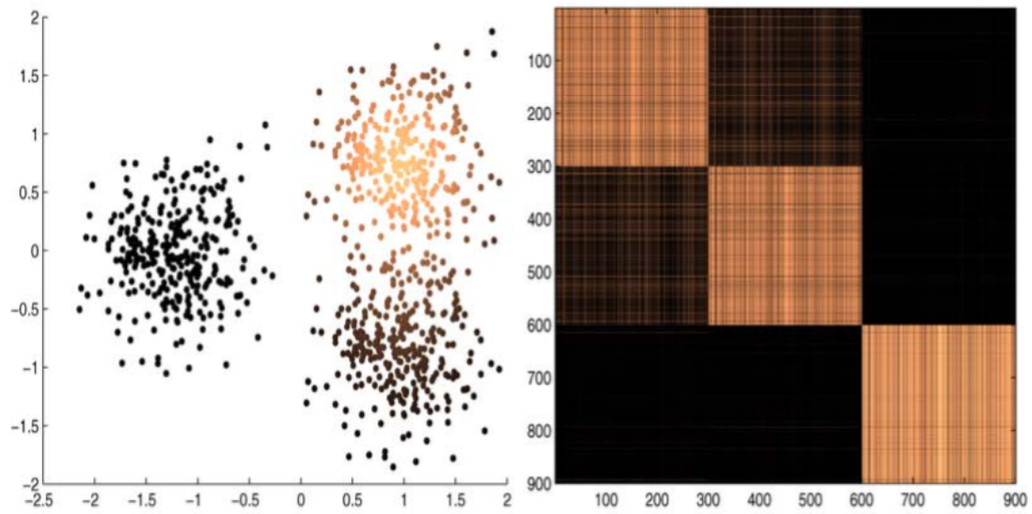
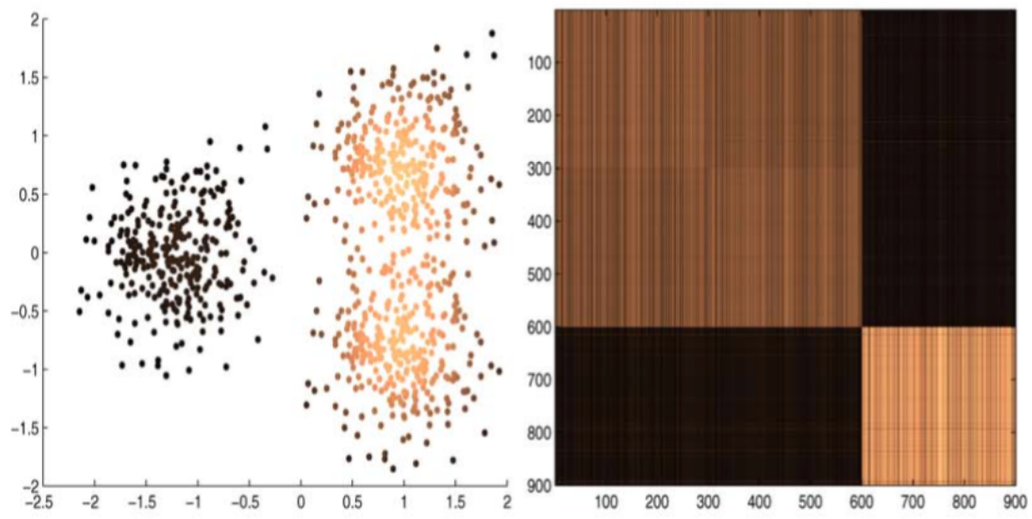


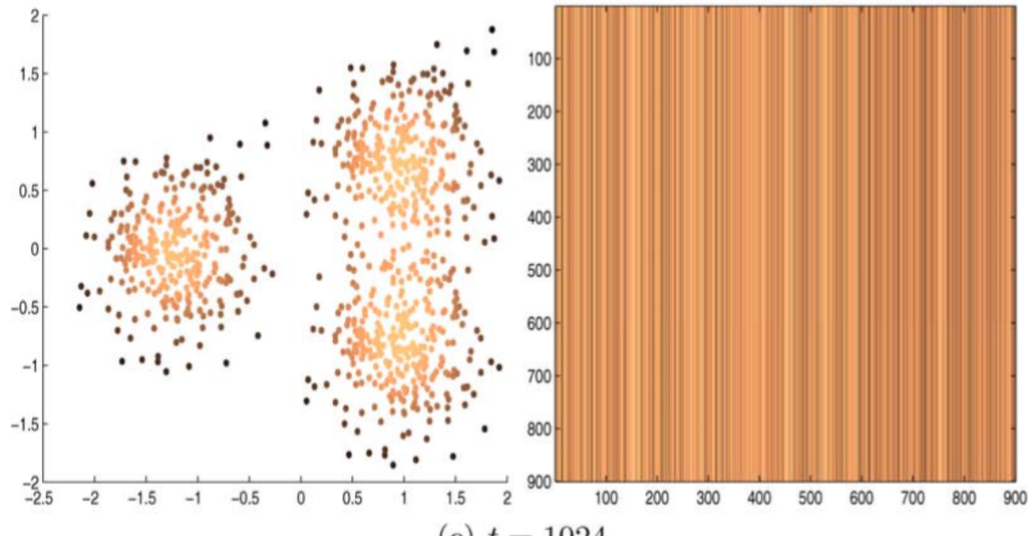
Fig. 4. From left to right: original curves, the densities of points, the embeddings via the graph Laplacian ($\alpha = 0$) and the embeddings via the Laplace–Beltrami approximation ($\alpha = 1$). In the latter case, the curve is embedded as a perfect circle and the arclength parametrization is recovered.



(a) $t = 8$



(b) $t = 64$



(c) $t = 1024$