Syllabus, Math 994-003

Computational Harmonic Analysis and Data Science
Spring 2018

Course Description: This course will cover aspects of modern computational harmonic analysis at the interface of signal processing and data science. A central theme of the course is to find good representations of functional data (e.g., time series, images, etc), where the quality of the representation is measured through notions of sparsity, characterization of certain functional classes, and eventually empirical data driven measures.

The prologue of the course will cover the rudiments of Fourier analysis and discrete signal processing. The shortcomings of the Fourier transform will motivate us to study localized time-frequency representations of functions, which will introduce the windowed Fourier transform as well as the continuous and dyadic wavelet transforms. Unlike the Fourier transform, which characterizes only the global regularity of a function, wavelet transforms characterize the local regularity of functions, and we will prove fundamental results along these lines. Windowed Fourier and wavelet transforms will be placed in a more general mathematical context via the study of redundant dictionaries and frame theory. Motivated in part by the sparsity of wavelet transforms, we will then aim to understand how to find sparse representations in general dictionaries, and at the conclusion of this part of the course look at recent methods that learn, in a data driven fashion, dictionaries of functions that yield sparse representations. Dictionary learning in turn leads to the study of more complicated learning models; convolutional neural networks are a natural place to turn. In the final part of the course we will study these networks, as well as mathematically tractable models for them (i.e. ones in which we can prove theorems) based upon nonlinear cascades of semi-discrete frame operators.

The primary textbook for the course will be *A Wavelet Tour of Signal Processing: The Sparse Way*, 3rd edition, by Stéphane Mallat. The course may also draw a bit of material from *Wavelets and Operators*, by Yves Meyer. The final part of the course on convolutional neural networks will be based on current papers in the field. All parts of the course (minus the prologue) will highlight current research and papers.

The course will assume knowledge of real analysis (Lebesgue integration, $\mathbf{L}^p$ spaces, Banach and Hilbert spaces).

Instructor Information

- **instructor:** Matthew Hirn
- **office:** 2507F, Engineering Building
- **email:** mhirn@msu.edu
- **phone:** (517) 432-0611

- **course webpage:** `https://matthewhirn.wordpress.com/teaching/spring-2018-math-994-003/`

Meeting Time and Location:

- Thursday 12:40 PM – 3:30 PM

- C100, Wells Hall

Office Hours:

- Tuesday, 1:00 PM – 3:00 PM

Prerequisites:

- Real analysis (Lebesgue integration, $\mathbf{L}^p$ spaces, Banach and Hilbert spaces).

Resources:

- *A Wavelet Tour of Signal Processing: The Sparse Way*, 3rd edition, by Stéphane Mallat (required)
- *Wavelets and Operators*, by Yves Meyer (we may use some parts, we will see, not required)
- *Harmonic Analysis and Applications*, by John Benedetto (detailed background on Fourier analysis, not required)
- Various papers on the mathematics of convolutional neural networks (specific ones to be determined)

Grading:

- Homework: 80%
- Attendance: 20% (This is a small class! So it is important that we all attend).

How the course will work:

We will use *A Wavelet Tour of Signal Processing* for the first 70-80% of the course (see the tentative course outline below). The final 20-30% will be on convolutional neural networks and will be based off of some recent papers in the field. We will not be able to cover every detail in lecture, so it will be up to you to read the book and papers outside of class to fill in gaps. These readings will be part of the exercises, and will be clearly noted in the course notes.

Exercises will be posted on the course website, and a subset of the (non-reading) exercises will be graded. After each class there could be anywhere from zero to several new exercises; generally they will be due one week after they are posted. Exercises will be a mix of mathematical proofs and some programming (in MATLAB). Each person is required to submit their own exercises, although you may discuss them with your classmates.

Attendance will be noted each class. Each unexcused absence will incur a 2% deduction from your final grade (maximum of 20%).

There will be no **Final Exam**.

Tentative **course outline:**

- Chapter 2: The Fourier Kingdom
- Chapter 3: Discrete Revolution
- Chapter 4: Time Meets Frequency

- Chapter 6: Wavelet Zoom
- Chapter 5: Frames
- Chapter 12: Sparsity in Redundant Dictionaries
- Convolutional Neural Networks based on Redundant Semi-Discrete Frames

Academic Honesty: Cheating in any form will not be tolerated and will be reported. You will receive a zero on any assignment in which there is a case of cheating. This includes, but is not limited to, plagiarism, failure to give proper citations, and copying another's work.

If you are preparing an assignment and have a question about whether you are adhering to this policy, please ask me. If you work on an assignment with other students, you must give credit to your collaborators.

MSU's policy on academic integrity can be found at the following URL:
`https://msu.edu/unit/ombud/academic-integrity/`

Disability Services: Accommodations for persons with disabilities can and will be made in this course. All arrangements will be organized through the RCPD office as MSU. Persons with disabilities who are interested in the available services should contact the MSU Resource Center for Persons with Disabilities (RCPD) at (517) 884-7273 or online at `http://www.rcpd.msu.edu`.